

## Graph-theoretic Approach to Optimal Synthesis of Supply Networks: Distribution of Gasoline from a Refinery

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### Abstract

The synthesis of a supply network is profoundly convoluted because of its combinatorial complexity. Global or even near optimal solutions are, more often than not, unobtainable through the heuristic methods. On the other hand, the majority of the algorithmic methods, which mainly resort to mixed integer programming, in theory, can give rise to global optima; however, they are effective only for relatively minute networks. Obviously, it is highly desirable that a novel paradigm be established for optimally synthesizing supply networks; the adoption of the graph-theoretic method based on P-graphs (process graphs) is proposed herein for the synthesis of optimal supply networks. The proposed method is illustrated with examples. Each example has yielded simultaneously the optimal and near optimal structures of the supply network performing a specific task in the ranked order. The example reveals a unique feature of the method.

**Keywords:** Supply network; graph-theoretic; P-graphs; optimal synthesis

### 1. Introduction

The optimal design, i.e., synthesis, of a supply network tends to be profoundly convoluted because of its combinatorial complexities. Thus, excessive time and effort are often required for formulation and computation except for some cases [1]. As a result, a limited number of papers has been published on this subject [2, 3]. Furthermore, the majority, if not all, of the published papers adopts algorithmic methods to carry out the optimization of algebraic models via mixed integer programming (MIP). An approach proposed herein for the optimal synthesis of supply networks resorts to the unique graph-theoretic method based on process graphs (P-graphs) originally developed for process-network synthesis [4-8]. The approach is demonstrated by applying it to the optimal supply-network design for a fuel product, i.e., gasoline, from a refinery.

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Gasoline produced from crude oil is conveyed through a supply network to markets; obviously, gasoline is a material for which the law of mass conservation is universally valid. This material can be regarded as the raw material entering into the network or the final product exiting from the network. Any actions imposed on or disturbances affecting the material conveyed through the network will induce a change in one or more of its attributes, such as static pressure, flowrate, lot size, and/or locations, thereby transforming the material. Upon reaching the markets or retailers through a series of these transformations, the material can be regarded as the final product exiting from the network. Thus, operating, i.e., functional, units can be unequivocally identified at locations of such actions or disturbances. Naturally, the networks can be represented as P-graphs. One of the two cornerstones of the current graph-theoretic method for process-network synthesis is obviously the representations of the operating units with these P-graphs. The other is a set of 5 axioms of combinatorially feasible process networks [5, 6]. These 5 axioms give rise to 3 highly efficient algorithms for implementing the method. The profound efficacy of the proposed approach is demonstrated with two examples of gasoline manufactured at a refinery and supplied to retailers through distribution networks containing various terminals.

## 2. Methodology

The present methodology comprises the following: (a) representing all the plausible operating units identified in terms of P-graphs; (b) composing the maximal structure from the P-graphs of the operating units via algorithm MSG (maximal structure generation); (c) generating exhaustively the combinatorially feasible network structures as solution-structures from the maximal structure via algorithm SSG (solution-structure generation); and (d) identifying all the feasible network structures among the combinatorially feasible network structures via MIP or, alternatively, determining only a limited number of the optimal and near optimal networks, in the ranked order of the objective function, directly from the maximal structure via algorithm ABB (accelerated branch and bound) [4-6, 9]

### 2.1. P-graph representations of operating units

The structure of a supply network, visualized as a process network, is represented by P-graphs, which are unique bipartite graphs. Unlike conventional bipartite graphs, or digraphs, the P-graphs are capable of capturing the syntactic and semantic contents of process networks. A P-graph comprises two types of vertices or nodes for representing materials and operating units; the former is symbolized by circles, and the latter, by horizontal bars. Table 1, to be elaborated later, illustrates the conventional as well as P-graph representations of operating units.

### 2.2. Implementation of algorithms

At the outset, the maximal structure of the gasoline-supply network, playing the role of process network, is composed via algorithm MSG with the P-graphs of all the operating units at its input. This algorithm totally excludes any combinatorially infeasible network structure in light of the five axioms in constituting a supply network. Thus, the maximal structure is the rigorous, minimally complex super-structure containing exclusively and exhaustively the combinatorially feasible network structures.

Subsequent to the generation of the maximum structure, the combinatorially feasible network structures are exhaustively recovered as the solution-structures by resorting to algorithm SSG. Each solution-structure signifies a combinatorially feasible network of pathways linking the raw materials to the products. Nevertheless, not all the solution-structures are necessarily feasible due to the violation of the mass balances in or around

the network. The feasibility of an individual solution-structure, i.e., combinatorially feasible network structure, is assessed by optimizing the objective function via MIP subject to the mass-balance constraints. Naturally, this also gives rise to the optimality of the individual feasible network structures.

In practice, only a limited number of optimal and near optimal structures would be of interest. Such network structures can be determined in the ranked order in terms of the objective function by means of algorithm ABB directly from the maximal structure. The objective function can be, profit, cost, sustainability, speed of supply, or any combination thereof.

**3. Illustration**

The proposed approach is illustrated with a supply network to deliver gasoline from a refinery to one or two terminals through various routes by two means of transportation, i.e., pipelines and tanker-trucks. The optimal and near-optimal networks in the ranked order of cost are obtained for the two examples, one depicted in Figure 1 and the other depicted in Figure 2. Conventional and P-graph representations of an operating unit identified are provided in Table 1 as an example.

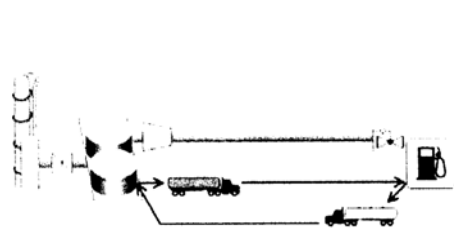


Figure 1 Gasoline supply from a refinery to a terminal (Example 1).

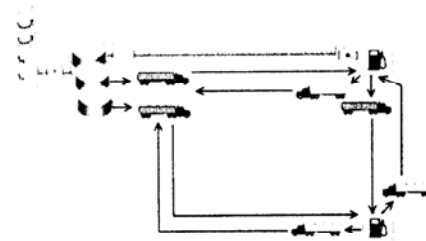


Figure 2 Gasoline supply from a refinery to two terminals (Example 2).

Table 1. Conventional and P-graph representations of an operating unit: gasoline loading on trucks

Operating units				Streams		
No	Designation	Function	Diagrammatic representation		Notation	Description
			Conventional	P-graph		
4	DT1	Loading of gasoline on trucks at distribution terminal 1	<p><math>DT1([G^1],[T]) = [G^1T]</math></p>		$S1-2 [G^1]$	Gasoline supplied to DT1
					$S5-2 [T]$	Trucks supplied to DT1
					$S4 [G^1T]$	Gasoline loaded on trucks

### 3.1. Example 1

Gasoline is transported from one refinery to a single terminal through a pipeline or by tanker-trucks; see Figure 1. The empty tanker-trucks are returned to the refinery to be reloaded for the succeeding delivery. The feasible network structures vary depending on the operating costs and capacity constraints of the pipeline and tanker-trucks. The resultant combinatorially feasible as well as feasible network structures are presented in Tables 2 and 3 for two sets of capacity constraints.

Table 2. Combinatorially feasible and feasible networks for Example 1

Combinatorially feasible networks {operating units}	Feasible networks {operating units}	Optimal value of objective function (\$/month)	Rank
N1 {1,2,3,6,f1}	N1* {1,2,3,6,f1}	29	1
N2 {1,2,3,4,5,7,f1}	N2* {1,2,3,7,f1}	31	2
N3 {1,4,5,8,f1}	N3* {1,4,5,8,f1}	34	3
N4 {1,2,3,4,5,6,7,f1}			
N5 {1,2,3,4,5,6,8,f1}			
N6 {1,2,3,4,5,7,8,f1}			
N7 {1,2,3,4,5,6,7,8,f1}			

Table 3. Combinatorially feasible and feasible networks for Example 1 (Min. flowrate of S1-1 and S1-2 = 1)

Combinatorially feasible networks {operating units}	Feasible networks {operating units}	Optimal value of objective function (\$/month)	Rank
N1 {1,2,3,6,f1}	N1* {1,2,3,6,f1}	29	1
N2 {1,2,3,4,5,7,f1}	N2* {1,2,3,4,5,7,f1}	34	2
N3 {1,4,5,8,f1}	N3* {1,4,5,8,f1}	34	2
N4 {1,2,3,4,5,6,7,f1}			
N5 {1,2,3,4,5,6,8,f1}	N5* {1,2,3,4,5,6,8,f1}	35	4
N6 {1,2,3,4,5,7,8,f1}			
N7 {1,2,3,4,5,6,7,8,f1}			

### 3.2. Example 2

Unlike Example 1, two terminals are involved in this example; however, gasoline is supplied to the second terminal only by tanker-trucks as depicted in Figures 2 and 3. Empty tanker-trucks from the two terminals are considered to belong to the same materials in the network. They are returned to the refinery to be reloaded for the succeeding delivery; 72 combinatorially feasible network structures have been

identified for this example, which, in turn, have yielded 4 feasible networks presented in Table 4.

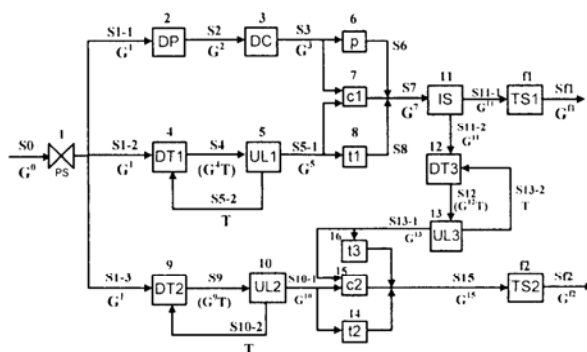


Figure 3 Process flowsheet of Example 2.

Table 4. Feasible networks for Example 2

Rank	Optimal value of objective function (\$/day)	Feasible networks {operating units}
1	103	N1* {1,2,3,6,11,12,13,16,f1,f2}
2	105	N2* {1,2,3,6,9,10,11,14,f1,f2}
3	110	N3* {1,4,5,8,9,10,11,14,f1,f2}
4	120	N4* {1,4,5,8,11,12,13,16,f1,f2}

**4. Discussion**

The efficacy of adopting the current graph-theoretic method based on P-graphs for systems, which are not traditionally regarded as process networks, such as supply networks, has been amply demonstrated by Halim and Srinivasan [10] in crafting the decision support system for waste minimization. The graph-theoretic method based on P-graphs definitely reveal the structural and operating features of supply networks in unquestionably more details than the conventional algorithmic method based on the MIP. Moreover, the superior computational efficiency of the former over the latter, especially for complex networks, has been unequivocally pointed out [8].

The efficacy of the proposed methodology is demonstrated with the two examples of a gasoline supply network. In Example 1, the combinatorially feasible solutions, i.e., networks, are identified via algorithms MSG and SSG [4-6]. The second columns of Tables 2 and 3 list the feasible networks determined via MIP for all combinatorially feasible networks contained in the first column of each table. Note that not all the combinatorially feasible networks are feasible; moreover, the number of feasible networks identified varies according to the mass constraints imposed as discernable in Tables 2 and 3. Specifically, Table 1 contains the results obtained without any mass flow constraint, and Table 2 contains the results obtained when the minimum mass

flowrates of streams S1-1 and S1-2 are constrained to be 1. In Example 2, only a limited number of feasible networks satisfying the production requirements are recovered as the optimal or near-optimal networks in the ranked order via algorithms MSG and ABB without resorting to algorithm SSG and MIP [4~6, 9].

It is of utmost importance to simultaneously generate some near-optimal supply networks in the ranked order of the objective-function values along with the optimal one. These near-optimal networks serve as the stand-bys to immediately replace the optimal network in case of interruption arising from man-made catastrophes, e.g., warfare, or natural catastrophes, e.g., wild fires or storms. Such capabilities are totally absent from the MIP-based approaches.

## 5. Conclusion

A highly efficient approach has been proposed for synthesizing, i.e., designing, a supply network for a fuel product. It has been unequivocally demonstrated that such a supply network is a process network, thereby rendering it possible to adopt the graph-theoretic method based on P-graphs (process graphs) for its synthesis. The method yields the optimal as well as near optimal networks simultaneously in the ranked order of a specific objective function, such as profit or cost. The profound efficacy of the proposed approach is amply demonstrated with two examples of supplying gasoline from a single refinery to one terminal in one example and two terminals in the other.

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