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Solving vehicle assignment problems by process-network synthesis to minimize cost and environmental impact of transportation

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Abstract A method and software are proposed for optimal assignment of vehicles to transportation tasks in terms of total cost and emission. The assignment problem is transformed into a process-network synthesis problem that can be algorithmically handled by the P-graph framework. In the proposed method, each task is given by a set of attributes to be taken account in the assignment; this is also the case for each vehicle. The overall mileage is calculated as the sum of the lengths of all the routes to be travelled during, before, after, and between the tasks (Desaulniers et al. 1998; Baita et al. 2000). Cost and emission are assigned to the mileages of each vehicle type. In addition to the globally optimal solution of the assignment problem, the P-graph framework provides the *n*-best suboptimal solutions that can be ranked according to multiple criteria. The viability of the proposed method is illustrated by an example.

Keywords P-graph · Combinatorial optimization · Vehicle assignment · Transportation

The variables used in this article include $T, S, A, t_s, t_e, l_s, l_e, d, l_a, c_t, e_t, v_{max}$—vehicle assignment problem; $P, R, O, a, c, U_c, L_p, U_p, l, u$—process-network synthesis (PNS) problem; m^*, o^*, x^*, z^*—optimal solution of PNS problem.

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List of Symbols

T	Set of tasks
S	Set of resources
$P_i \in T$	Trip i to be performed
$t_s(P_i)$	Starting time of trip i
$l_s(P_i)$	Starting location of trip i
$t_e(P_i)$	Ending time of trip i
$l_e(P_i)$	Ending location of trip i
d	Distance for each pair of locations
$R_k \in S$	Vehicle k
$l_a(R_k)$	Actual location of the vehicle k
$c_t(R_k)$	The cost of vehicle k
$e_t(R_k)$	The CO_2 emission of vehicle k
$v_{max}(R_k)$	The maximum speed of vehicle k
$A(P_i)$	The set of resources potentially capable of performing task P_i
P	The set of the final targets to be achieved
R	The set of the initially available resources
M	The set of entities
m_j	entity j
$o_i = (\alpha_i, \beta_i)$	Activity i with α_i set of preconditions and β_i set of targets
O	The set of candidate activities
L_{p_j}	Lower bound on the gross result
U_{p_j}	Upper bound on the gross result
U_{c_j}	Upper bound on gross utilization
u_i	Upper bound for the volume of activity o_i
l_i	Lower bound for the volume of activity o_i
cm_j	Price for each resource on target
cp_i	Proportional constant of activity i
cf_i	Fixed charge of activity i
a_{ji}	The difference between the production and consumption rate of entity m_j by activity o_i
m^*	Set of entities in the optimal structure

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o^*	Set of activities in the optimal structure
x^*	The vector of the optimal volumes of activities
z^*	Objective value of the optimal solution

Introduction

According to EPA reports, transportation accounted for approximately 29% of the total U.S. greenhouse gas (GHG) emissions in 2006. Moreover, transportation is the fastest growing source of U.S. GHGs, corresponding to 47% of the net increase in total U.S. emissions since 1990. Transportation is also the largest end-use source of CO_2 (Ilyas et al. 2010). Nevertheless, these estimates do not include emissions from additional processes involved in the lifecycle of the transportation systems, such as the extraction and refining of fuel and manufacture of vehicles, which are also significant sources of domestic and international GHG emissions (US EPA 2009).

The two major means for reducing the pollution caused by the transportation are changing the personal behavior (Aizura et al. 2010) of the end users and improving the quality of transporting networks (Atkins et al. 2009; Crilly and Zhelev 2010), for which the economic crisis of these years serves as a motivation.

Optimization software for supporting human decisions is essential to implement the two major means mentioned above (Perry et al. 2007; Tan and Foo 2009; Klemes et al. 2010). Presented herein is an algorithmic method for calculating the optimal assignment of vehicles to transportation tasks supported by software tools at each step.

Problem definition

The vehicle assignment problem of interest is defined by a set T of tasks, a set S of resources, and related parameters. For each $P_i \in T$ task, i.e., trip to be performed, the starting time $t_s(P_i)$ and location $l_s(P_i)$ as well as the ending time $t_e(P_i)$ and location $l_e(P_i)$ are given.

The actual location $l_a(R_k)$, cost $c_t(R_k)$, CO_2 emission $e_t(R_k)$, and maximum speed $v_{max}(R_k)$ are specified for each $R_k \in S$ resource, i.e., vehicle. Moreover, set $A(P_i)$ lists those resources potentially capable of performing task P_i, for each $P_i \in T$. Each task is considered to require the full capacity of a single resource. Tables 1 and 2 contain the details of the example with three tours and three vehicles. The distances between pairs of the locations in the example are indicated in Fig. 1.

Methodology

The vehicle assignment problem is transformed into the corresponding process-network synthesis (PNS) problem. The P-graph representation of the PNS problem provides an easily discernable structural model and the basis for effective solution by combinatorial accelerations as well (Friedler et al. 1992, 1993, 1995, 1996; Varbanov and Friedler 2008; Lam et al. 2010). The proposed approach ensures that the resultant solution is globally optimal; in addition, it yields the *n*-best feasible assignments in the ranked order.

For a set M of entities, a PNS problem is given by a triplet (P, R, O), where the set $P \subseteq M$ contains the final targets to be reached; set $R \subseteq M$ contains the initially available resources; and set $O \subseteq \wp(M) \times \wp(M)$, comprising the candidate activities for forming a network to reach each of the final targets by deploying any of the available resources. Each activity o is defined by the pair of its preconditions and outcomes, i.e., for each $o \in O$: $o = (\alpha, \beta)$ where $\alpha, \beta \subseteq M$. A precondition can be the availability of a resource or an outcome of another activity. In any transportation network, the location and time of availability

Table 1 Tasks to be completed for the example

Task $P_i \in T$	Starting time $t_s(P_i)$	Starting location $l_s(P_i)$	Ending time $t_e(P_i)$	Ending location $l_e(P_i)$	Potential resources $A(P_i)$
P1	7:00	Tihany	7:40	Almadi	R1, R2
P2	8:00	Veszprem	9:20	Fehervar	R1, R3
P3	8:40	Veszprem	9:40	Tihany	R1, R2

Table 2 Available resources for the example

Resources $R_k \in S$	Location $l_a(R_k)$	Cost $c_t(R_k)$ (€/km)	CO_2 emission $e_t(R_k)$ (g/km)	Maximum speed $v_{max}(R_k)$ (km/h)
R1	Tihany	0.6	375	90
R2	Fehervar	0.5	400	90
R3	Kenese	0.4	300	90

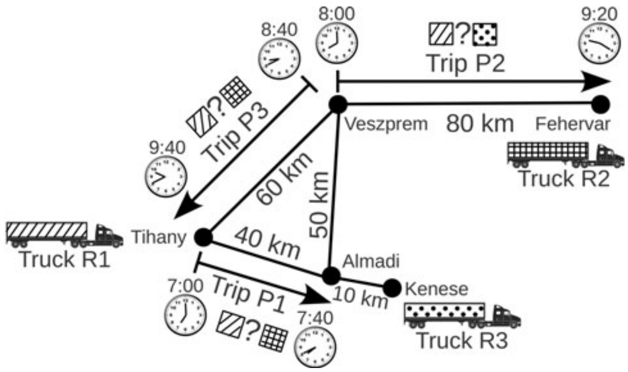

Fig. 1 Map of the tasks, resources, and locations for the example

of resources and outcomes are essential, and thus, they are well defined. For each vehicle assignment problem, what follows is either specified or defined: The tasks to be completed serve as the final targets of the network in set *P*; the resources are listed in set *R*; and set *O* of candidate activities involves the tasks performed by each appropriate resource or movement of resources from one location to another. Figure 2 shows algorithm VAPtoPNS for

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input: (T, S, A, t_s, t_e, l_s, l_e, d, l_a, c_t, e_t, v_max) assignment problem
output: (𝒫, ℛ, 𝒪, a, c, U_c, L_p, U_p, l, u) parametric process synthesis problem
begin
    𝒫 := T; ℛ := S ∪ {EUR, CO2}; 𝒪 := ∅;
    U_{c_EUR} := ∞; c_EUR := 1; U_{c_CO2} := ∞; c_CO2 := 0;
    for all R_k ∈ S do
        U_{c_{R_k}} := 1, c_{R_k} := 0;
        Travel_R_k := ({EUR, CO2}, {km_R_k});
        𝒪 := 𝒪 ∪ {Travel_R_k};
        l_{Travel_R_k} := 0, u_{Travel_R_k} := ∞;
        a_{EUR,Travel_R_k} := −c_t(R_k); a_{CO2,Travel_R_k} := −e_t(R_k);
        a_{km_R_k,Travel_R_k} := 1;
    end for
    q := 1;
    for all P_i ∈ T do
        L_{p_{P_i}} := 1, U_{p_{P_i}} := 1, c_{P_i} := 0;
        for all R_k ∈ A(P_i) do
            O_q := ({R_k, km_R_k}, {R_k_l_s(P_i)_t_s(P_i)});
            𝒪 := 𝒪 ∪ {O_q}; q := q + 1;
            l_{O_q} := 0, u_{O_q} := ∞;
            a_{R_k,O_q} := −1; a_{km_R_k,O_q} := −d(l_a(R_k), l_s(P_i));
            a_{R_k_l_s(P_i)_t_s(P_i),O_q} := 1;
            P_i_by_R_k := ({R_k_l_s(P_i)_t_s(P_i), km_R_k}, {R_k_l_e(P_i)_t_e(P_i), P_i});
            𝒪 := 𝒪 ∪ {P_i_by_R_k};
            l_{P_i_by_R_k} := 1, u_{P_i_by_R_k} := 1;
            a_{R_k_l_s(P_i)_t_s(P_i),P_i_by_R_k} := −1; a_{km_R_k,P_i_by_R_k} := −d(l_s(P_i), l_e(P_i)
            a_{R_k_l_e(P_i)_t_e(P_i),P_i_by_R_k} := 1; a_{P_i,P_i_by_R_k} := 1;
        end for
        for all P_j ∈ T do
            for all R_k ∈ A(P_i) ∩ A(P_j) do
                if t_e(P_i) ≤ t_s(P_j) and d(l_e(P_i), l_s(P_j)) / (t_s(P_j) − t_e(P_i)) ≤ v_max(R_k) then
                    O_q := ({R_k_l_e(P_i)_t_e(P_i), km_R_k}, {R_k_l_s(P_j)_t_s(P_j)});
                    𝒪 := 𝒪 ∪ {O_q}; q := q + 1;
                    l_{O_q} := 0, u_{O_q} := ∞;
                    a_{R_k_l_e(P_i)_t_e(P_i),O_q} := −1; a_{km_R_k,O_q} := −d(l_e(P_i), l_s(P_j));
                    a_{R_k_l_s(P_j)_t_s(P_j),O_q} := 1;
                end if
            end for
        end for
    end for
end
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Fig. 2 Algorithm for transforming a vehicle assignment problem into the corresponding process-network synthesis problem

constructing the corresponding process synthesis (PNS) problem for a vehicle assignment problem (VAP). It generates three classes of activities (Barany et al. 2010); in the first class, an activity is assigned to the movement of each resource from its initial location to each task, which can be performed by the resource; in the second class, an activity is assigned to each task–resource pair, where the task can potentially be completed by the resource; and finally, in the third class, activities are defined for the movement of the resources between two tasks. The third class of activities occur if and only if both tasks can be performed by a single activity; the first task precedes the second task; and the resource can travel to from the ending location of the first task to the starting location of the second task in time by its maximal speed. Tables 3, 4, and 5 list the resources, targets, and candidate activities, respectively, generated by the algorithm VAPtoPNS to be considered in process synthesis for the example.

Beside structural information, optimization entails the definition of a set of parameters. A parametric definition of the PNS problem involves the following: the lower bound L_{p_j} on the gross result is greater than zero for each m_j final target; and it is equal to zero for any other entity. Thus,

$$L_{p_j} = \begin{cases} > 0, \forall m_j \in P \\ 0, \text{otherwise} \end{cases} \quad (1)$$

The upper bound, U_{p_j}, on overall result for each m_j resource is equal to zero, and greater than or equal to L_{p_j} for any other entity, i.e.,

Table 3 Resources to be considered in process synthesis for the example

Resource R_j	Upper bound $U_c R_j$	Cost C_{R_j}
R1	1	0
R2	1	0
R3	1	0
EUR	∞	1
CO2	∞	0

Table 4 Targets to be considered in process synthesis for the example

Target P_j	Lower bound L_{P_j}	Upper bound U_{P_j}
P1	1	1
P2	1	1
P3	1	1

Table 5 Activities to be considered in process synthesis for the example

Activity o_i	Precondition a_{ji} m_j for all $m_j \in \alpha_i$	Result a_{ji} m_j for all $m_j \in \beta_i$	Lower bound l_i	Upper bound u_i
Travel_R1	0.6 EUR, 375 CO2	km_R1	0	∞
Travel_R2	0.5 EUR, 400 CO2	km_R2	0	∞
Travel_R3	0.4 EUR, 300 CO2	km_R3	0	∞
O1	R2, 80 km_R2	R2_Veszprem_0840	0	∞
O2	R2, 140 km_R2	R2_Tihany_0700	0	∞
O3	R1	R1_Tihany_0700	0	∞
O4	R1, 60 km_R1	R1_Veszprem_0840	0	∞
O5	R1, 60 km_R1	R1_Veszprem_0800	0	∞
O6	R3, 60 km_R3	R3_Veszprem_0800	0	∞
O7	R2_Almadi_0740, 50 km_R2	R2_Veszprem_0840	0	∞
O8	R1_Almadi_0740, 50 km_R1	R1_Veszprem_0840	0	∞
P1ByR1	R1_Tihany_0700, 40 km_R1	R1_Almadi_0740, P1	1	1
P1ByR2	R2_Tihany_0700, 40 km_R2	R2_Almadi_0740, P1	1	1
P2ByR1	R1_Veszprem_0800, 80 km_R1	R1_Fehervar_0920, P2	1	1
P2ByR3	R3_Veszprem_0800, 80 km_R3	R3_Fehervar_0920, P2	1	1
P3ByR1	R1_Veszprem_0840, 60 km_R1	R1_Tihany_0940, P3	1	1
P3ByR2	R2_Veszprem_0840, 60 km_R2	R2_Tihany_0940, P3	1	1

$$U_{p_j} = \begin{cases} 0, \forall m_j \in R \\ \geq L_{p_j}, \text{otherwise} \end{cases} \tag{2}$$

The upper bound, U_{c_j}, on gross utilization of a m_j resource is greater than zero, and equal to zero for any other entity, i.e.,

$$U_{c_j} = \begin{cases} > 0, \forall m_j \in R \\ 0, \text{otherwise} \end{cases} \tag{3}$$

Also given are the lower bound, l_i, and the upper bound, u_i, for the volume of each activity o_i and the price, cm_j, for each resource or target. The cost of an activity is estimated by a linear function of its volume with a fixed charge. For the cost function of each activity, the proportionally constant cp_i and the fixed charge cf_i are defined. Note that the objectives in PNS problems corresponding to vehicle assignment problems, including the costs and emissions of transportations, are represented as resources. Thus, cost parameters of the activities are equal to zero. Relations between entities and activities are denoted by a_{ji} which gives the difference between the production and consumption rate of entity m_j by activity o_i. Furthermore, in the optimal structure let $m^* \subseteq M$ denote the set of entities; $o^* \subseteq O$, the set of activities; x^*, the vector of the optimal volumes of activities for the problem; and z^*, its objective value. The aim is to determine the network (m^*, o^*, x^*, z^*) satisfying the following conditions (Eqs. 4–10); z^*, minimal as given in Eq. 11 below

$$\forall o_i = (\alpha_i, \beta_i) \in o^* : m_j \in \alpha_i \Leftrightarrow a_{ji} < 0, \quad m_j \in \beta_i \Leftrightarrow a_{ji} > 0 \tag{4}$$

$$m^* = \bigcup_{(\alpha_i, \beta_i) \in o^*} \alpha_i \cup \beta_i \tag{5}$$

$$x^* = \left[x_1^*, x_2^*, \ldots, x_n^*\right]^T \tag{6}$$

$$0 < x_i^*, l_i \leq x_i^* \leq u_i \Leftrightarrow o_i \in o^* \tag{7}$$

$$\forall m_j \in m^* \cap R : -U_{c_j} \leq \sum_{o_i \in o^*} a_{ji} x_i \leq 0 \tag{8}$$

$$\forall m_j \in m^* \cap P : L_{p_j} \leq \sum_{o_i \in o^*} a_{ji} x_i \leq U_{p_j} \tag{9}$$

$$\forall m_j \in m^* \backslash R \backslash P : 0 \leq \sum_{o_i \in o^*} a_{ji} x_i \leq U_{p_j} \tag{10}$$

$$z^* = \sum_{(\alpha_i, \beta_i) = o_i \in o^*} \left(\mathrm{cf}_i + x_i^* * \left(\mathrm{cp}_i - \sum_{m_j \in \alpha \cup \beta} a_{ji} \mathrm{cm}_j \right) \right) \tag{11}$$

In the P-graph framework, algorithm MSG gives rise to the maximal structure for the PNS problem (Friedler et al. 1993). This maximal structure serves as the input to the generation and solution of the mathematical model by algorithm ABB (Friedler et al. 1996). Figure 3 depicts the maximal structure for the example. Algorithm ABB yields the optimal and a finite number of n-best suboptimal networks in the ranked order. Table 6 contains the

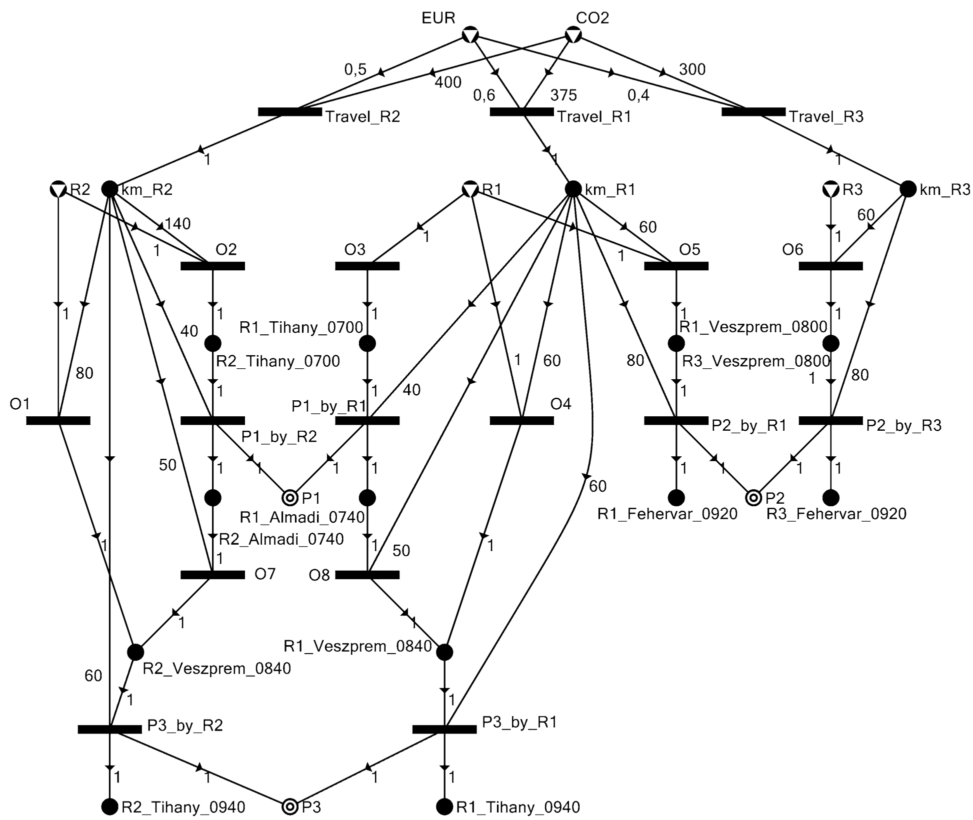

Fig. 3 P-graph representation of the maximal structure for the example

Table 6 Five best vehicle assignments for the example

Solution	Assignments			Mileage (km)			Total mileage (km)	Cost (€)	CO_2 emission (kg)
	P1	P2	P3	R1	R2	R3			
#1	R1	R3	R1	150	0	140	290	146	98.25
#2	R1	R3	R2	40	140	140	320	150	113.00
#3	R2	R3	R2	0	290	140	430	201	158.00
#4	R2	R3	R1	120	180	140	440	218	159.00
#5	R2	R1	R2	140	290	0	430	229	168.50

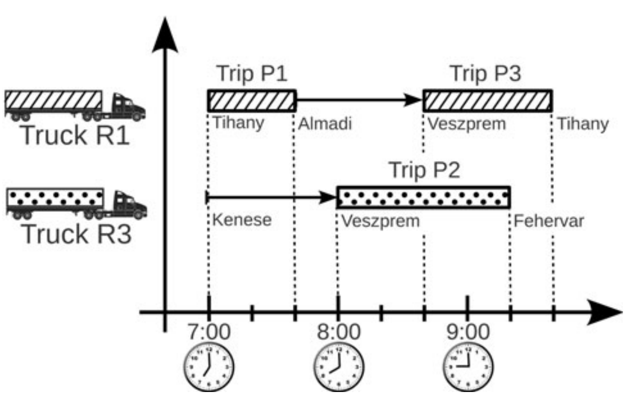

Fig. 4 Optimal assignment of vehicles to trips for the example

five best vehicle assignments for the example. Algorithms MSG and ABB have been executed by software PNS Studio (P-graph.com 2010).

Results and discussion

The alternative feasible solutions generated by algorithm ABB can be ranked according to multiple criteria. Such criteria can be, e.g., the costs and emissions of the vehicles. Even if both cost and emission are usually related to the mileage of a vehicle, the optimal or *n*-best assignments may differ for these two objectives due to the reduced emissions from modern vehicles of high costs; see Solutions #4 and #5 for the example in Table 6.

Note that the optimal solution is identical from the economical and ecological points of view as depicted in detail in the Gannt diagram in Fig. 4. In contrast, Table 6 highlights that the rankings of assignments by their costs and emissions result in different orders, and thus, it is important to generate alternative assignments.

Conclusions

An algorithmic method has been proposed for generating optimal and *n*-best suboptimal solutions for a vehicle assignment problem. The method has been crafted by reformulating an assignment problem as a PNS problem and solving the resultant problem by algorithms and software of the P-graph framework. The potential of the proposed method has been illustrated by solving an example in which the optimal and suboptimal alternative assignments of the different ranked order emerge by regarding the costs and emissions of the vehicles as the objective function.

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