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INTEGRATED SYNTHESIS OF A PROCESS AND ITS FAULT-TOLERANT CONTROL SYSTEM

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ABSTRACT

The previously developed method for integrated process and control system (IPCS) synthesis (Hangos *et al.*, 1994) has been extended to the cases for which process safety or reliability is of primary concern. An unambiguous representation of the resultant integrated process and fault-tolerant control system structure, IPFCS structure in brief, is rendered possible through the CP-graph originally proposed for IPCS synthesis. A new axiom for the fault-tolerant controllability is established; it states that more than one independent control path are required for each controlled variable. The fundamental combinatorial algorithm of IPCS synthesis, i.e., algorithm CMSG, has been revised so that the modified set of axioms is capable of identifying the controllable, fault-tolerant structures of an IPFCS synthesis problem. The efficacy of the proposed method for IPFCS synthesis method is demonstrated with a relatively simple example by systematically varying the level of the fault tolerance.

KEYWORDS

Process synthesis, structural controllability, fault-tolerance, combinatorial methods.

INTRODUCTION

An important and yet extremely difficult task confronting the process-control community is to establish a fully integrated methodology for performing simultaneous syntheses of processes and their control systems essentially from the outset of process design. In recent years, the foundation of a graph-theoretic approach to process synthesis has been well established (Friedler *et al.*, 1993); meanwhile, the structural controllability of a process has been successfully analyzed on the basis of digraph-type process models (Reinschke, 1988). Collectively, these two developments have given rise to a graph-theoretic method for integrated process and control system (IPCS) synthesis (Hangos *et al.*, 1994). A set of axioms is obtained for describing the combinatorially feasible and controllable structures by resorting to the notion of CP-graph. The maximal controllable structure of an IPCS synthesis problem has been defined as the union of combinatorially feasible and controllable IPCS structures containing the optimal IPCS structure.

For an increasing number of processes, safety in general and fault-tolerance in particular are of utmost concern. For such processes, the integration of fault-tolerance requirements into the early stages of process design, i.e., in the phase of process synthesis, is of great importance. Thus, our method for IPCS synthesis is extended to construct the present method for integrated process and fault-tolerant control system (IPFCS) synthesis.

STRUCTURE REPRESENTATION

The major concepts and terminologies of IPFCS synthesis, analogous to those of IPCS synthesis of Hangos *et al.* (1994), are given in this section. For this purpose, let $\mathcal{M}$ and $\mathcal{A}$ be two disjoint finite sets where $\mathcal{M}$ is the set of materials and $\mathcal{A}$ is the set of actuators.

Definition 1. *Operating unit* o_i on set $\mathcal{M}$ of materials and set $\mathcal{A}$ of actuators is defined to be the quintuple, $o_i = (\alpha_i, \beta_i, \gamma_i, F_i, \pi_i)$, where α_i is the set of input materials ($\alpha_i \subseteq \mathcal{M}$); β_i, the set of output materials ($\beta_i \subseteq \mathcal{M}$); γ_i, the set of possible actuators; F_i, the *performance* function of the operating unit; and π_i, the set of additional parameters, e.g., the costs. Note that the domain of F_i, dom(F_i), is a subset of $\alpha_i \cup \gamma_i$, and the range of F_i is a subset of the power set of β_i; thus, $F_i \subseteq (\alpha_i \cup \gamma_i) \times (\wp(\beta_i))$. If $F_i(x) = m$ for some $x \in \alpha_i$, i.e., $m \subseteq \beta_i$, then a change in material x effects a change in each element of m but not in any element of $\beta_i \setminus m$. Similarly, if $F_i(a) = m'$ for some $a \in \gamma_i$, i.e., $m' \subseteq \beta_i$, then actuator a may induce changes in each element of m', but not in any element of $\beta_i \setminus m'$.

Suppose that $\mathcal{O}$ is a set of operating units on sets $\mathcal{M}$ and $\mathcal{A}$. Let us now define mappings ψ^i, ψ^o, and ψ^a on the set of operating units as $\psi^i(o_j) = \alpha_j$, $\psi^o(o_j) = \beta_j$, and $\psi^a(o_j) = \gamma_j$ for operating unit $o_j = (\alpha_j, \beta_j, \gamma_j, F_j, \pi_j) \in \mathcal{O}$. Mappings ψ^i, ψ^o, and ψ^a are called the *input*, *output*, and *actuator selectors* of the operating units, respectively. Let us define similarly the input, output, and actuator selectors of a subset of the set of operating units by

$$\Psi^i(\mathcal{O}') = \bigcup_{o_j \in \mathcal{O}'} \psi^i(o_j) \quad \Psi^o(\mathcal{O}') = \bigcup_{o_j \in \mathcal{O}'} \psi^o(o_j) \quad \Psi^a(\mathcal{O}') = \bigcup_{o_j \in \mathcal{O}'} \psi^a(o_j)$$

respectively, where $\mathcal{O}' \subseteq \mathcal{O}$.

Definition 2. Triplet $(\mathcal{M}', \mathcal{A}', \mathcal{O}')$ defines a *CP-graph* on sets $\mathcal{M}$, $\mathcal{A}$, and $\mathcal{O}$ if $\mathcal{M}' \subseteq \mathcal{M}$ and $\mathcal{A}' \subseteq \mathcal{A}$ are a finite set of materials and that of actuators, respectively. It is supposed that $\mathcal{O}' \subseteq \mathcal{O}$ is a set of operating units on sets $\mathcal{M}'$ and $\mathcal{A}'$, i.e., $\psi^i(o_j)$, $\psi^o(o_j) \subseteq \mathcal{M}'$ for each $o_j \in \mathcal{O}'$; moreover, $\mathcal{A}' \subseteq \Psi^a(\mathcal{O}')$. CP-graph $(\mathcal{M}', \mathcal{A}', \mathcal{O}')$ has two *components*, i.e., *structural* component $(\mathcal{M}', \mathcal{A}', \mathcal{O}')^S$ and *control* component $(\mathcal{M}', \mathcal{A}', \mathcal{O}')^C$. The former, $(\mathcal{M}', \mathcal{A}', \mathcal{O}')^S$, is a directed bipartite graph where the set of its vertices is $\mathcal{M}' \cup \mathcal{O}'$, and the set of its arcs is

$$\{(x, o_i): o_i \in \mathcal{O}' \ \&\ x \in \psi^i(o_i)\} \cup \{(o_i, x): o_i \in \mathcal{O}' \ \&\ x \in \psi^o(o_i)\}.$$

The latter, $(\mathcal{M}', \mathcal{A}', \mathcal{O}')^C$, is a simple directed graph where the set of its vertices is $\mathcal{M}' \cup \mathcal{A}'$, and the set of its arcs is

$$\{(a, b): \text{there is an } o_i \in \mathcal{O}' \text{ such that } a \in \text{dom}(F_i) \cap (\mathcal{M}' \cup \mathcal{A}') \text{ and } b \in F_i(a)\}.$$

Note that the structural component of CP-graph $(\mathcal{M}', \mathcal{A}', \mathcal{O}')$ can be considered as P-graph $(\mathcal{M}', \mathcal{O}')$ (see, e.g., Friedler *et al.*, 1992). The set of actuators, the performance function, and the additional parameters are, however, irrelevant for the P-graph. Vertices of CP-graph $(\mathcal{M}', \mathcal{A}', \mathcal{O}')$ belonging to set $\mathcal{M}'$ are the vertices of the $\mathcal{M}$ type, and those belonging to set $\mathcal{O}'$ are the vertices of the $\mathcal{O}$ type.

Definition 3. The *union of the two CP-graphs*, $(\mathcal{M}', \mathcal{A}', \mathcal{O}')$ and $(\mathcal{M}'', \mathcal{A}'', \mathcal{O}'')$, is defined to be $(\mathcal{M}' \cup \mathcal{M}'', \mathcal{A}' \cup \mathcal{A}'', \mathcal{O}' \cup \mathcal{M}'')$. CP-graph $(\mathcal{M}', \mathcal{A}', \mathcal{O}')$ is defined to be the subgraph of CP-graph $(\mathcal{M}'', \mathcal{A}'', \mathcal{O}'')$ if $\mathcal{M}' \subseteq \mathcal{M}''$, $\mathcal{A}' \subseteq \mathcal{A}''$, and $\mathcal{O}' \subseteq \mathcal{O}''$.

Definition 4. Sequence a_1, a_2, ..., and a_n, is a *structural path in CP-graph* $(\mathcal{M}', \mathcal{A}', \mathcal{O}')$ from a_1 to a_n if it is a path of its structural component; it is denoted by $[a_1, a_n]^S$. Similarly, sequence b_1, b_2, ..., and b_m, is a *control path in CP-graph* $(\mathcal{M}', \mathcal{A}', \mathcal{O}')$ from b_1 to b_n if it is a path of its control component; it is denoted by $[b_1, b_m]^C$.

For CP-graph $(\mathcal{M}', \mathcal{A}', \mathcal{O}')$, let us define function Γ as a mapping from set $\mathcal{A}'$ to the set of natural numbers. Here, function Γ is termed as the *multiplicity function* of the actuators of the CP-graph (or simply, multiplicity function); it expresses the actual multiplicity of each actuator represented in CP-graph $(\mathcal{M}', \mathcal{A}', \mathcal{O}')$.

Definition 5. Quadruple ($\mathcal{M}'$, $\mathcal{A}'$, $\mathcal{O}'$, Γ) is called an *fCP structure* if ($\mathcal{M}'$, $\mathcal{A}'$, $\mathcal{O}'$) is a CP-graph, and Γ is a corresponding multiplicity function.

Example 1. o_1 and o_2 are operating units on set $\mathcal{M}^{(1)}$ = {A, B, C, D, E, F} of materials and set $\mathcal{A}^{(1)}$ = {a_1, a_2, a_3} of actuators where o_1 = ({A, B}, {C, D}, {a_1, a_2}, (B, {D}), (a_1, {C, D}), (a_2, {D})}, $\varnothing$) and o_2 = ({D}, {E, F}, {a_3}, {(D, {E, F}), (a_3, {F})}, $\varnothing$). Since F_1(B) = {D}, a change in material B effects a change in material D. A change in material B, however, does not induce any change in the other output material of operating unit o_1, i.e. C. Actuator a_1 of this operating unit may induce a change both in materials C and D since $F_1(a_1)$ = {C, D}. The multiplicity of actuators a_1, a_2, and a_3 is 2, 1 and 2, respectively. Therefore, $\Gamma(a_1)$=2, $\Gamma(a_2)$=1, and $\Gamma(a_3)$=2. Note that in Fig. 1, an operating unit is represented by a horizontal bar, —; a material by a circle, ●; and an actuator by a square, □. A number in this square is the value of multiplicity function Γ. In this figure, the material flows are denoted by solid arcs, and the control effects are indicated by dashed arcs.

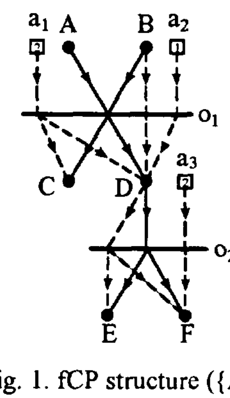


Fig. 1. fCP structure ({A, B, C, D, E, F}, {a_1, a_2, a_3}, {o_1, o_2}, {(a_1, 2), (a_2, 1), (a_3, 2)})

COMBINATORIALLY FEASIBLE AND FAULT-TOLERANTLY CONTROLLABLE PROCESS STRUCTURES

In the IPCS synthesis, a structure considered to be controllable, if there is at least one control path to a specific material represented in the structure [see axiom (SC1) in Hangos *et al.*, 1994]. To increase the fault-tolerancy of a controllable structure, a minimum number of such paths is required for each of these specific materials. For this purpose, let us introduce parameter r termed the *fault-tolerancy level*. Let us also introduce parameter r_a which is the maximal multiplicity of any actuator. Thus, the maximal value of the multiplicity function, Γ, must be less or equal to r_a, i.e., $\max_{a\in\mathcal{A}'}\Gamma(a) \le r_a$.

Let $\mathcal{P}\subseteq\mathcal{M}$ be the set of products and $\mathcal{R}\subseteq\mathcal{M}$ be the set of raw materials. Furthermore, let $\mathcal{O}$ be a set of operating units on set $\mathcal{M}$ of materials and set $\mathcal{A}$ of actuators. Then, sextuple ($\mathcal{P}$, $\mathcal{R}$, $\mathcal{O}$, $\mathcal{A}$, r, r_a) defines the combinatorial part of an IPFCS synthesis problem provided that $\mathcal{P}\cap\mathcal{R} = \varnothing$.

Axioms of combinatorially feasible structures. fCP structure ($\mathcal{M}'$, $\mathcal{A}'$, $\mathcal{O}'$, Γ) is a *combinatorially feasible structure* of IPFCS synthesis problem ($\mathcal{P}$, $\mathcal{R}$, $\mathcal{O}$, $\mathcal{A}$, r, r_a) if ($\mathcal{M}'$, $\mathcal{A}'$, $\mathcal{O}'$) is a CP-graph; moreover, it satisfies the following axioms.

(S1) Every final product is represented in the graph, i.e., $\mathcal{P}\subseteq\mathcal{M}'$.

(S2) A vertex of the $\mathcal{M}$-type has no input if and only if it represents a raw material, i.e., $\forall x\in\mathcal{M}'$, $\varphi^i(x) = \varnothing$ if and only if $x\in\mathcal{R}$.

(S3) Every vertex of the $\mathcal{O}$-type represents an operating unit defined in the IPFCS synthesis problem, i.e., $\mathcal{O}'\subseteq\mathcal{O}$.

(S4) Every vertex of the $\mathcal{O}$-type has at least one structural path leading to a vertex of the $\mathcal{M}$-type representing a final product, i.e., $\forall y_0\in\mathcal{O}'$, $\exists$ path $[y_0, y_1]^S$ where $y_1\in\mathcal{P}$.

(S5) If a vertex of the $\mathcal{M}$-type belongs to the graph, it must be an input to or output from at least one vertex of the $\mathcal{O}$-type in the graph, i.e., $\forall x\in\mathcal{M}'$, $\exists o\in\mathcal{O}'$ such that $x\in\psi^i(o)\cup\psi^o(o)$.

Axioms of structural fault-tolerant controllability. fCPstructure ($\mathcal{M}'$, $\mathcal{A}'$, $\mathcal{O}'$, Γ) satisfies the axioms of *structural fault-tolerant controllability for IPFCS synthesis problem* ($\mathcal{P}$, $\mathcal{R}$, $\mathcal{O}$, $\mathcal{A}$, r, r_a) if it satisfies the following two axioms.

(SFC1) The multiplicity of any actuator is not greater than r_a, i.e., $\max_{a \in \mathcal{A}'} \Gamma(a) \leq r_a$.

(SFC2) There exists at least r different control paths to each material $x \in \mathcal{M}'$ such that x is a product or is produced and also consumed by some operating units, i.e., $\forall x \in (\Psi^i(\mathcal{O}') \cap \Psi^o(\mathcal{O}')) \cup \mathcal{P}$ and $\mathcal{A}'(x) = \{aj : [aj, x]^C \text{ is a control path}\}$, $\sum_{aj \in \mathcal{A}'(x)} \Gamma(aj) \geq r$.

Definition 6. Structure ($\mathcal{M}'$, $\mathcal{A}'$, $\mathcal{O}'$, Γ) is a *combinatorially feasible and fault-tolerantly controllable* (CFFC) structure of IPFCS synthesis problem ($\mathcal{P}$, $\mathcal{R}$, $\mathcal{O}$, $\mathcal{A}$, r, r_a) on $\mathcal{M}$ if it satisfies axioms (S1) through (S5) and axioms (SFC1) and (SFC2).

Maximal fault-tolerantly controllable structure. In solving an IPFCS synthesis problem, the search for the optimal structure can be confined to the set of CFFC structures. Hence, those and only those operating units which are the members in at least one CFFC structure need be considered in developing a mathematical model of the problem, e.g., a MINLP model.

Let S($\mathcal{P}$, $\mathcal{R}$, $\mathcal{O}$, $\mathcal{A}$, r, r_a) be the set of CFFC structures for IPFCS synthesis problem ($\mathcal{P}$, $\mathcal{R}$, $\mathcal{O}$, $\mathcal{A}$, r, r_a); moreover, let us suppose that S($\mathcal{P}$, $\mathcal{R}$, $\mathcal{O}$, $\mathcal{A}$, r, r_a) $\neq \varnothing$.

Definition 7. The *maximal fault-tolerantly controllable structure* of IPFCS synthesis problem ($\mathcal{P}$, $\mathcal{R}$, $\mathcal{O}$, $\mathcal{A}$, r, r_a) is the union of all CP-graphs ($\mathcal{M}'$, $\mathcal{A}'$, $\mathcal{O}'$) for which there exists a multiplicity function Γ such that the structure ($\mathcal{M}'$, $\mathcal{A}'$, $\mathcal{O}'$, Γ) is a CFFC structure. The maximal fault-tolerantly controllable structure is denoted by μ($\mathcal{P}$, $\mathcal{R}$, $\mathcal{O}$, $\mathcal{A}$, r, r_a), i.e.,

$$\mu(\mathcal{P},\mathcal{R},\mathcal{O},\mathcal{A},r,r_a) = \bigcup_{(\mathcal{M}',\mathcal{A}',\mathcal{O}',\Gamma) \in S(\mathcal{P},\mathcal{R},\mathcal{O},\mathcal{A},r,r_a)} (\mathcal{M}',\mathcal{A}',\mathcal{O}')$$

Note that the maximal fault-tolerantly controllable structure is always a subgraph of the maximal structure of the corresponding IPCS synthesis problem.

CASE STUDY

To compare the results of the integrated process and control system synthesis with those with different fault-tolerant controllability considerations, Example 2 in Friedler *et al.* (1993) is extended with actuators and control paths as described in Hangos *et al.* (1994).

Example 2. Actuators e.g., manipulable valves, are placed at operating units to which raw materials are fed. Moreover, let us assume that all output materials from an operating unit are controllable by any of the input materials of this operating unit. In this case a class of IPFCS synthesis problems can be given by ($\mathcal{P}^{(2)}$, $\mathcal{R}^{(2)}$, $\mathcal{O}^{(2)}$, $\mathcal{A}^{(2)}$, r, r_a) where

$\mathcal{M}^{(2)}$ = {A, B, C, D, E, F, G, H, I, J, K, L, M, N, Q, T, U, V}, $\mathcal{P}^{(2)}$ = {B}, $\mathcal{R}^{(2)}$ = {F, H, M, T},

$\mathcal{A}^{(2)}$ = {a_1, a_4, a_5, a_6, a_8, a_{10}},

$\mathcal{O}^{(2)}$ = { ({C, D, F}, {A}, {a_1}, {(C, {A}), (D, {A}), (F, {A}), (a_1, {A})}, ∅), ({D}, {B, G}, ∅, {(D, {B, G})}, ∅), ({E}, {B, U}, ∅, {(E, {B, U})}, ∅), ({F, G}, {C, D}, {a_4}, {(F, {C, D}), (G, {C, D}), (a_4, {C, D})}, ∅), ({G, H}, {D}, {a_5}, {(G, {D}), (H, {D}), (a_5, {D})}, ∅), ({H, I}, {E}, {a_6}, {(H, {E}), (I, {E}), (a_6, {E})}, ∅), ({J, K}, {E}, ∅, {(J, {E}), (K, {E})}, ∅), ({M}, {G}, {a_8}, {(M, {G}), (a_8, {G})}, ∅), ({N, Q}, {H}, ∅, {(N, {H}), (Q, {H})}, ∅), ({T, U}, {I}, {a_{10}}, {(T, {I}), (U, {I}), (a_{10}, {I})}, ∅), ({V}, {J}, ∅, {(V, {J})}, ∅) }

The following two tables summarize the CFFC structures for two related IPFCS synthesis problems $(\mathcal{P}^{(2)}, \mathcal{R}^{(2)}, \mathcal{O}^{(2)}, \mathcal{A}^{(2)}, 2, 1)$ and $(\mathcal{P}^{(2)}, \mathcal{R}^{(2)}, \mathcal{O}^{(2)}, \mathcal{A}^{(2)}, 3, 2)$. The CP-graphs of these CFFC structures are shown in Fig. 2.; note that CP-graph 7 is the maximal fault-tolerantly controllable structure in both cases. The multiplicity functions of the solution structures are also given in Tables 1 and 2, where "N/A" stands for non-applicable, i.e. the actuator is not represented in the CP-graph. Obviously, several CFFC structures are based on the same CP-graph (see, e.g., CP-graph #3 in Table 1).

Table 1. CFFC structures for $(\mathcal{P}^{(2)}, \mathcal{R}^{(2)}, \mathcal{O}^{(2)}, \mathcal{A}^{(2)}, 2, 1)$

#	CP-graph	Γ				
	#	a_4	a_5	a_6	a_8	a_{10}
1	1	1	N/A	N/A	1	N/A
2	2	N/A	1	N/A	1	N/A
3	3	1	0	N/A	1	N/A
4	3	0	1	N/A	1	N/A
5	3	1	1	N/A	0	N/A
6	3	1	1	N/A	1	N/A
7	4	N/A	N/A	1	N/A	1
8	5	1	N/A	1	1	1
9	6	N/A	1	1	1	1
10	7	1	0	1	1	1
11	7	0	1	1	1	1
12	7	1	1	1	0	1
13	7	1	1	1	1	1

Note that CFFC structure #6 is also appropriate for IPFCS problem $(\mathcal{P}^{(2)}, \mathcal{R}^{(2)}, \mathcal{O}^{(2)}, \mathcal{A}^{(2)}, 3, 1)$.

Table 2. CFFC structures for $(\mathcal{P}^{(2)}, \mathcal{R}^{(2)}, \mathcal{O}^{(2)}, \mathcal{A}^{(2)}, 3, 2)$

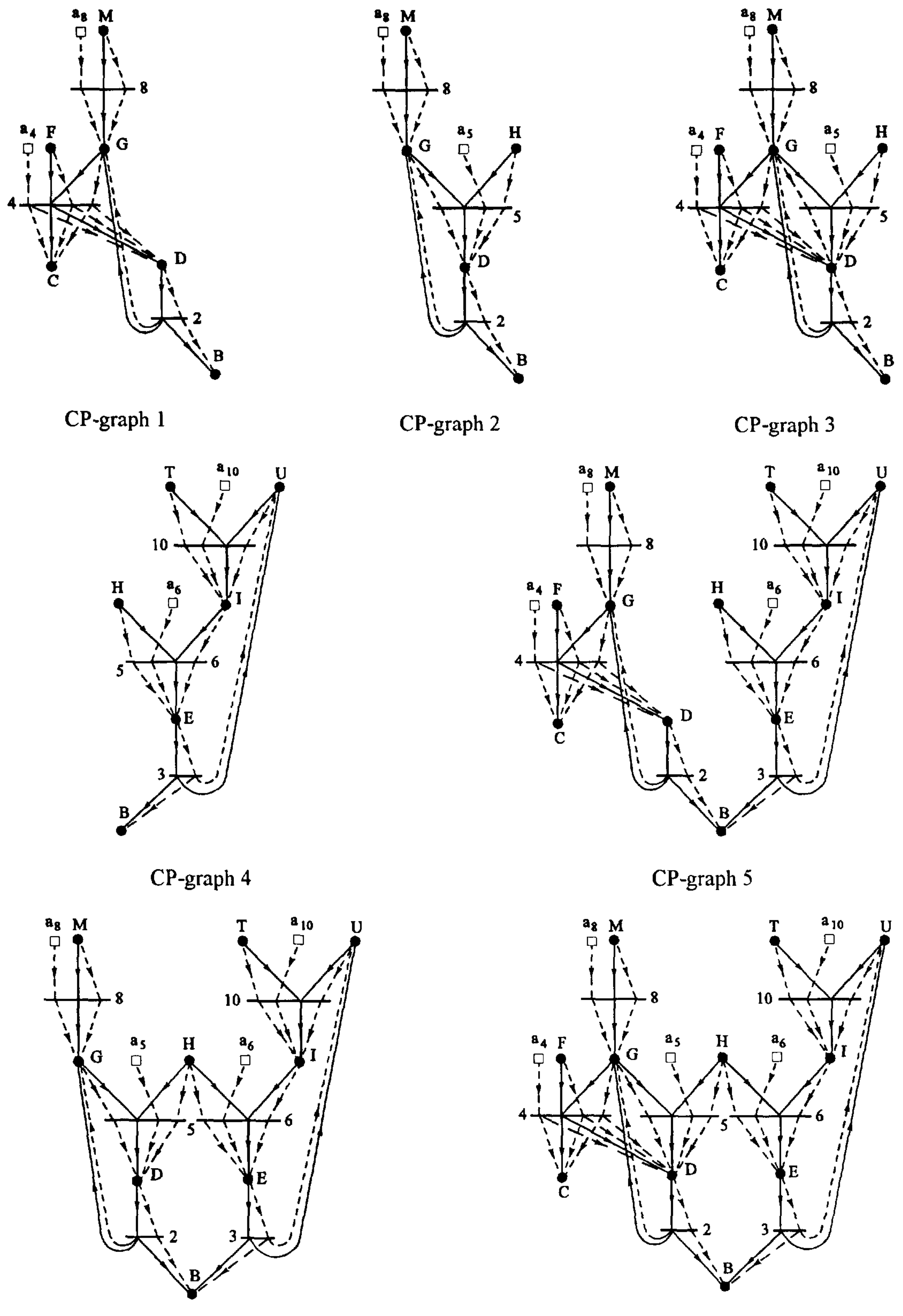
#	CP-graph	Γ				
	#	a_4	a_5	a_6	a_8	a_{10}
1	1	1	N/A	N/A	2	N/A
2	1	2	N/A	N/A	1	N/A
3	1	2	N/A	N/A	2	N/A
..	..	..	..	..	..	..
94	7	2	2	2	2	1
95	7	2	2	2	2	2

REFERENCES

Friedler, F., K. Tarjan, Y. W. Huang, and L. T. Fan, Graph-Theoretic Approach to Process Synthesis: Polynomial Algorithm for Maximal Structure Generation, *Comput. chem. Engng.* 17(9), 929-942 (1993).

Reinschke, K. J., *Multivariable Control. A Graph-Theoretic Approach,* In: Lecture Notes in Control and Information Sciences, 108 (Eds: M. Thoma and A. Wyner). Springer Verlag (1988).

Hangos, K. M., F. Friedler, J. B. Varga, and L. T. Fan, A Graph-Theoretic Approach to Integrated Process and Control System Synthesis, In: *Prep. IFAC Workshop on Integration of Process Design and Control*, Baltimore, U.S.A. (1994).



CP-graph 1 CP-graph 2 CP-graph 3

CP-graph 4 CP-graph 5

CP-graph 6 CP-graph 7

Fig. 2. CP-graphs of the IPFCS synthesis problems