Design of Optimal and Near-Optimal Enterprise-Wide Supply Networks for Multiple Products in the Process Industry

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The optimal and near-optimal enterprise-wide networks are designed, that is synthesized, for supplying feedstocks and distributing multiple products manufactured from these feedstocks in the process industry by resorting to the graph-theoretic method based on process graphs (P-graphs). Such feedstocks and products, conveyed through supply networks, are invariably materials for which the law of mass conservation is universally valid. Moreover, any of the actions applied to or exerted on a given feedstock or product, transiting through the supply networks, will induce a change in at least one of its attributes, thereby transforming the feedstock or product. Examples of the actions are loading, blending, pumping, tracking, unloading, subdividing, and/or wrapping; and those of the attributes are chemical composition, physical state, flow characteristics, external appearance, and/or location. Thus, in the broadest sense, any supply network can be regarded as a process network. The feedstocks and the products manufactured from them serve as the raw materials for and the products from the supply network at its entrance and exit, respectively. An operating, that is functional, unit can be unequivocally identified where any action is exerted on these raw materials or products. Naturally, the networks can be represented graph-theoretically as P-graphs. The proposed method is illustrated with an example involving three process plants, three markets, and three products under the three scenarios of coordination, cooperation, and competition. It has given rise simultaneously to the optimal as well as near-optimal supply networks in the ranked order. The example, formulated as the mixed integer linear programming problem, yields the same optimal solutions only, but not the near-optimal solutions in the ranked order.

1. Introduction

The design, or synthesis, of optimal supply networks entails much time and effort for formulation and computation[1] that can probably be attributed to its inordinate combinatorial complexity. This is especially the case for the networks supplying multiple commodities through multiple distribution tiers. Consequently, relatively little has been published on the optimal design of supply networks.[2,3] Moreover, the majority, if not all, of the limited number of contributions available on the subject resorts to algorithmic methods executing the optimization of algebraic models via mixed integer programming (MIP). Particularly noteworthy among them is that by Ryu and Pistikopoulos.[4] It deals with the design, or synthesis, of optimal enterprise-wide, complex supply networks in the process industry involving multicommodities. Ryu and Pistikopoulos[4] have appropriately and rigorously formulated their supply network synthesis problem as a process-network synthesis problem. Such a synthesis problem is also the topic of the current contribution. In contrast to their work, however, the current contribution resorts to the unique graph-theoretic method based on process graphs (P-graphs), which has originally been developed strictly for process-network synthesis.[5−7]

The feedstocks and products, conveyed through supply networks, are invariably materials for which the law of mass conservation is universally valid. These feedstocks and the products serve as the raw materials for and products from the supply networks at their entrances and exits, respectively. Any actions imposed on or disturbances affecting a stream of a given feedstock or product, or simply a material conveyed through the supply networks, will induce a change in one or more of its attributes, thereby transforming this material. Operating, that is functional, units can be unequivocally identified at locations of such actions or disturbances. Moreover, various constraints arising from the different operating strategies can be expressed as additional functional (operating) units in the networks. Naturally, the networks can be represented graph-theoretically as P-graphs (Appendix A in the Supporting Information). One of the two cornerstones of the current graph-theoretic, or combinatorial, method for process-network synthesis is obviously the representations of the operating units with the P-graphs. The other cornerstone is a set of five axioms.[7] These axioms are simply the restatement of the definitions of the raw material or product as well as the totally natural consequences of the law of mass conservation. The axioms give rise to three highly efficient algorithms for implementing the method, which include algorithms MSG (maximal structure generation), SSG (solution-structure generation), and ABB (accelerated branch and bound) (Appendices B−D in the Supporting Information).

The profound efficacy of the current method is demonstrated with an example taken from the contribution of Ryu and Pistikopoulos.[4] It involves three process plants, three markets, and three products under the scenarios of coordination, cooperation, and competition.

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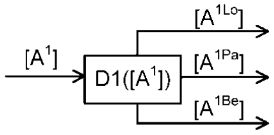
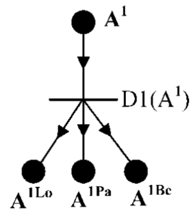
Table 1. Conventional and P-graph Representations of an Operating Unit: Distribution of a Product

Operating units					Materials	
No.	Designation	Function	Diagrammatic representation: Conventional	Diagrammatic representation: P-graph	Notation	Description
3	D1	Distribution of the product manufactured at Plant 1	 **D1([A^1])** **= ([A^{1Lo}], [A^{1Pa}], [A^{1Be}])**		A^1	Product manufactured at Plant 1
					A^{1Lo} A^{1Pa} A^{1Be}	Products manufactured at Plant 1 for markets Lo, Pa, and Be

2. Methods

The current method is capable of optimally designing a network for supplying the feedstocks to plants as well as for delivering the multiple products manufactured at the plants. It involves the followings:[5−7] (a) representing all the plausible operating units identified in terms of P-graphs; (b) composing the maximal structure from the P-graphs of the operating units via algorithm MSG; (c) generating exhaustively the combinatorially feasible network structures as solution-structures from the maximal structure via algorithm SSG; and (d) identifying all the feasible network structures among the combinatorially feasible network structures via MIP, or alternatively, determining only a limited number of the optimal and near-optimal networks, in the ranked order of the objective function, directly from the maximal structure via algorithm ABB.

P-Graph Representations of Operating Units. The structure of a supply network is represented by P-graphs, which are unique bipartite graphs. Unlike conventional bipartite graphs or digraphs, the P-graphs are capable of capturing the syntactic and semantic contents of process networks. A P-graph comprises two types of vertices or nodes for representing materials and operating units; the former is symbolized by circles, and the latter, by horizontal bars. Table 1 illustrates the conventional as well as the P-graph representations of an operating unit.

Implementation of Algorithms. At the outset, the maximal structure of the supply network of concern is composed via algorithm MSG with the P-graphs of all of the operating units at its input. In light of the five axioms, this algorithm totally excludes any combinatorially infeasible network structure in constituting a supply network. Thus, the maximal structure is the rigorous superstructure with minimum complexity containing exclusively and exhaustively the combinatorially feasible network structures.

Subsequent to the generation of the maximum structure, the combinatorially feasible network structures are exhaustively recovered as the solution-structures by resorting to algorithm SSG. Each solution-structure provides a network of pathways linking the raw materials to the products. Nevertheless, not all of the solution-structures are necessarily feasible due to the violation of the mass balances in or around the network. The feasibility of an individual solution-structure, that is a combinatorially feasible network structure, is assessed by optimizing the objective function via MIP subject to the mass-balance constraints. Naturally, this also gives rise to the optimality of the individual feasible network structures.

In practice, only a limited number of optimal and near-optimal structures would be of interest. Such network structures can be determined in the ranked order in terms of the objective function by means of algorithm ABB directly from the maximal structure. The objective function can be profit, cost, sustainability, or speed of supply, or any combination thereof.

Once the functional units are defined for the supply chain of interest, the implementation of the algorithms can be automatically performed with the software developed for process-network synthesis.

3. Illustration

The proposed methodology is illustrated with a supply network involving three plants, three markets, and three products, as shown in Figure 1. Largely based on the parameters

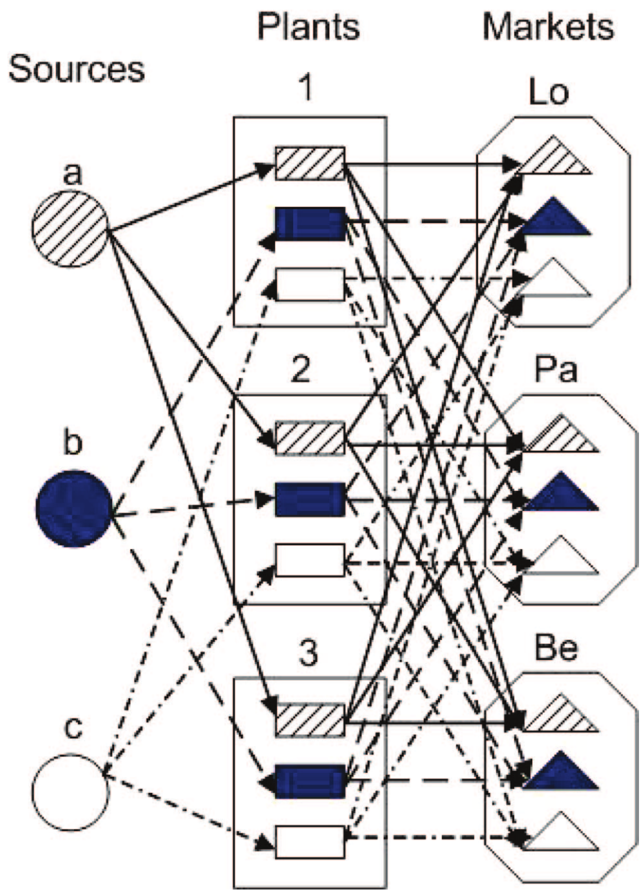


Figure 1. Flowsheet of the supply network for three plants supplying three markets with three products.[4]

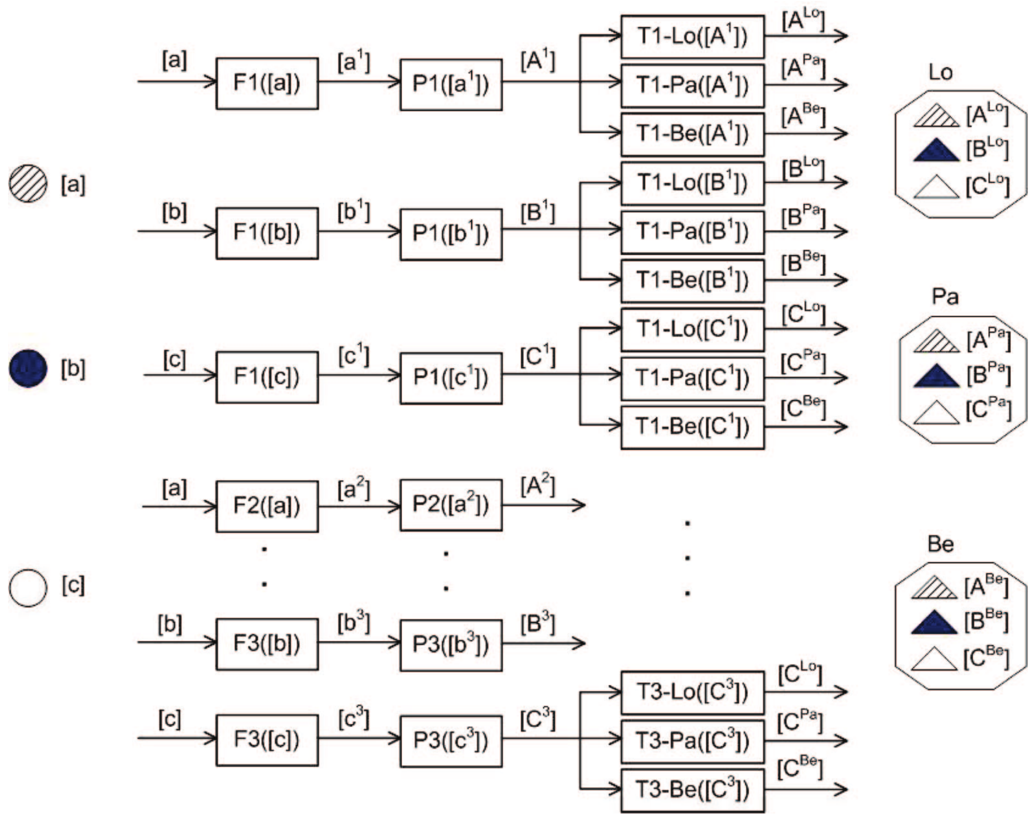


Figure 2. Functional, that is operating, units in scenario I.

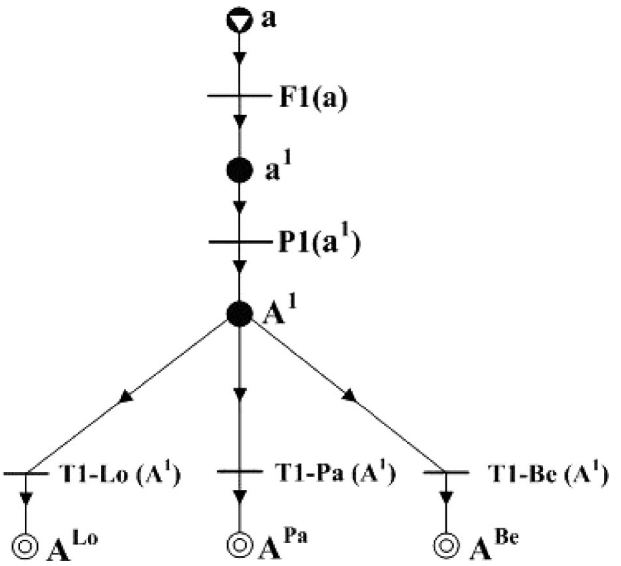


Figure 3. P-graph representation for manufacturing product A at plant 1 for markets Lo, Pa, or Be in scenario I.

given in the aforementioned contribution of Ryu and Pistikopoulos (Appendix E in the Supporting Information),[4] the optimal and near-optimal networks in the ranked order of cost are obtained for three scenarios. Conventional and P-graph representations are provided in the tabular form for all the operating (functional) units and materials identified in scenario II (Appendix F in the Supporting Information). Such representations can be readily extracted from those tabulated for scenario I or crafted additionally for scenario III.

Scenario I. Three plants supply markets Lo, Pa, and Be with products A, B, and C according to the demand of each market. Figure 2 exhibits the individual supply chains comprising the functional, that is operating, units identified. These units include those for feeding the feedstocks to the plants; manufacturing the products at the plants; and delivering or transporting the products from the plants to the markets. Figure 3 describes the corresponding P-graph representation for part of the flowsheet in which product A is manufactured at plant 1 for markets Lo, Pa, or Be.

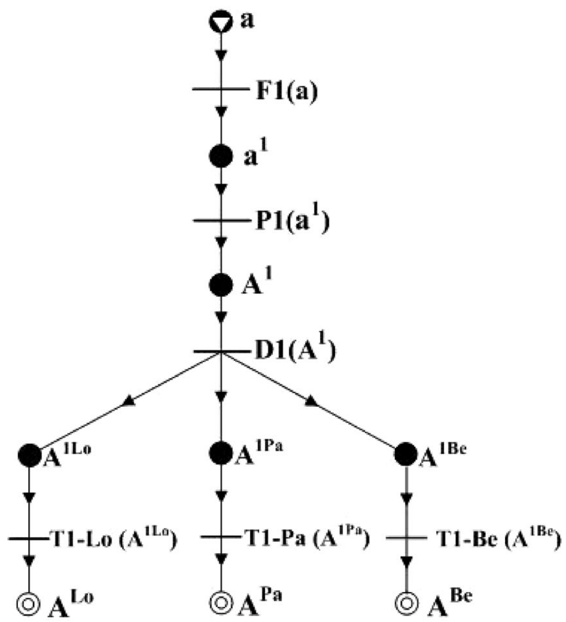


Figure 4. P-graph representation for manufacturing product A at plant 1 for markets Lo, Pa, and Be in scenario II.

Scenario II. This scenario requires each plant to manufacture only one product to be supplied to all three markets. This entails the amount of this product to be at least as large as the sum of

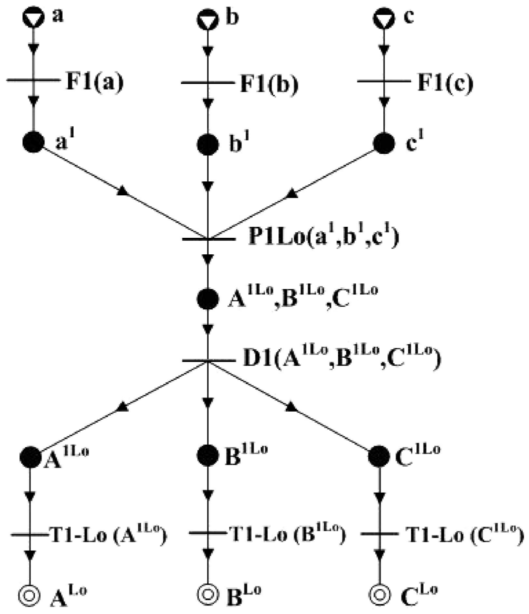


Figure 5. P-graph representation for manufacturing products A, B, and C at plant 1 for market Lo in scenario III.

market demands. The distribution unit in Table 1 signifies the coordination of the deliveries of this product to all three markets. Figure 4 displays the P-graph representing the manufacture of product A at plant 1.

Scenario III. Each plant manufactures all three products for a single market. In other words, the available feedstocks to this plant need to exceed the demand by any single market. The coordination of the deliveries of the three products to a market is represented as a distribution unit. Figure 5 depicts the part of the P-graph representing the manufacture of products A, B, and C at plant 1 for market Lo for this scenario.

The maximal structure for scenario II is presented in Figure 6 as an example. The optimal and the near-optimal networks obtained with the proposed methodology are listed in Tables 2, 3, and 4, along with the optimal network obtained with the conventional algorithmic method resorting to MIP.

4. Discussion

The undertaking of the present work is motivated by the fact that the proposed method, differing substantially from the currently deployed methods, can be applied to the design of supply networks with ease. Comparatively speaking, the graph-theoretic method based on P-graphs proposed herein is computationally efficient as well as capable of revealing the structural details of the network; ensuring the global optimality of the most superior network constituted; and rendering the collection of networks generated robust. The last two attributes are due to the fact that the method generates the optimal as well as the near-optimal networks in the ranked order of the objective (or cost) function. The difference is finite, but not infinitesimal, between the objective-function values of any pair of successively inferior networks, thus indicating the globality of the most superior, that is best, network. Moreover, the networks generated collectively constitute a robust system: the second-best network serves as the back-up for the best, that is optimal, network; the third-best network, for the second-best network; and so on.

The efficacy of adopting the current graph-theoretic method based on P-graphs for systems, not traditionally regarded as process networks, such as supply networks, has been amply demonstrated by Halim and Srinivasan[8] in developing the decision support system for waste minimization. It appears that the current graph-theoretic method based on P-graphs definitely reveals the structural and operating features of supply networks in substantially more details than the conventional algorithmic method resorting to MIP. Last but not least, the superior computational efficiency of the former over the latter especially for complex networks, has been unequivocally pointed out;[9] it has also been repeatedly validated.[10−12]

The efficacy of the proposed method is demonstrated with an enterprise-wide supply network.[4] At the outset, the best ten optimal solutions are identified in the ranked order under each scenario, via algorithms MSG and ABB. Nevertheless, only those solutions satisfying the production requirements are retained as the optimal or near-optimal solutions under all three scenarios. It is worth noting that the optimal solution obtained in each scenario is identical to that obtained by MIP.[4]

In scenario I of demanding three plants to supply three markets with three products, the resultant optimal solution indicates that all three products are to be manufactured only at plants 1 and 3. Moreover, the second-best solution implies that plant 1 is to manufacture products A and C; plant 3 is to manufacture only product B; and plant 2 remains idle. Nevertheless, the difference in terms of the total profits is only 0.09% between the best and the second-best solutions.

In scenario II of requiring each plant to manufacture a single product for all three markets, the resultant optimal solution assigns product A to plant 1; product C to plant 2; and product B to plant 3; the total profit for this optimal solution is 2.8% less than that of scenario I. Furthermore, the second-best solution assigns product A to plant 1; product B to plant 2; and product

Table 2. Optimal and Near-Optimal Solutions Obtained by the MILP and the Graph-Theoretic Approach for Scenario I[a]

Supplying Quantity, $X_{l,d,i}$ (ton[b])		MILP			P-graph								
		Optimal			Optimal			2nd best			3rd best		
		Lo	Pa	Be	Lo	Pa	Be	Lo	Pa	Be	Lo	Pa	Be
plant 1	A	600	400	500	600	400	500	600	400	500	600	400	500
	B												
	C											600	
plant 2	A												
	B							400	400	600			
	C	600	600	400	600	600	400						
plant 3	A												
	B	400	400	600	400	400	600				400	400	600
	C							600	600	400	600		400
total profit ($/ton)			412 149			412 149			411 777			411 508	

[a] Notes: Scenario I corresponds to the competition according to policy (c) in Ryu and Pistikopoulos.[4] [b] The quantities are interpreted on a unit time basis.

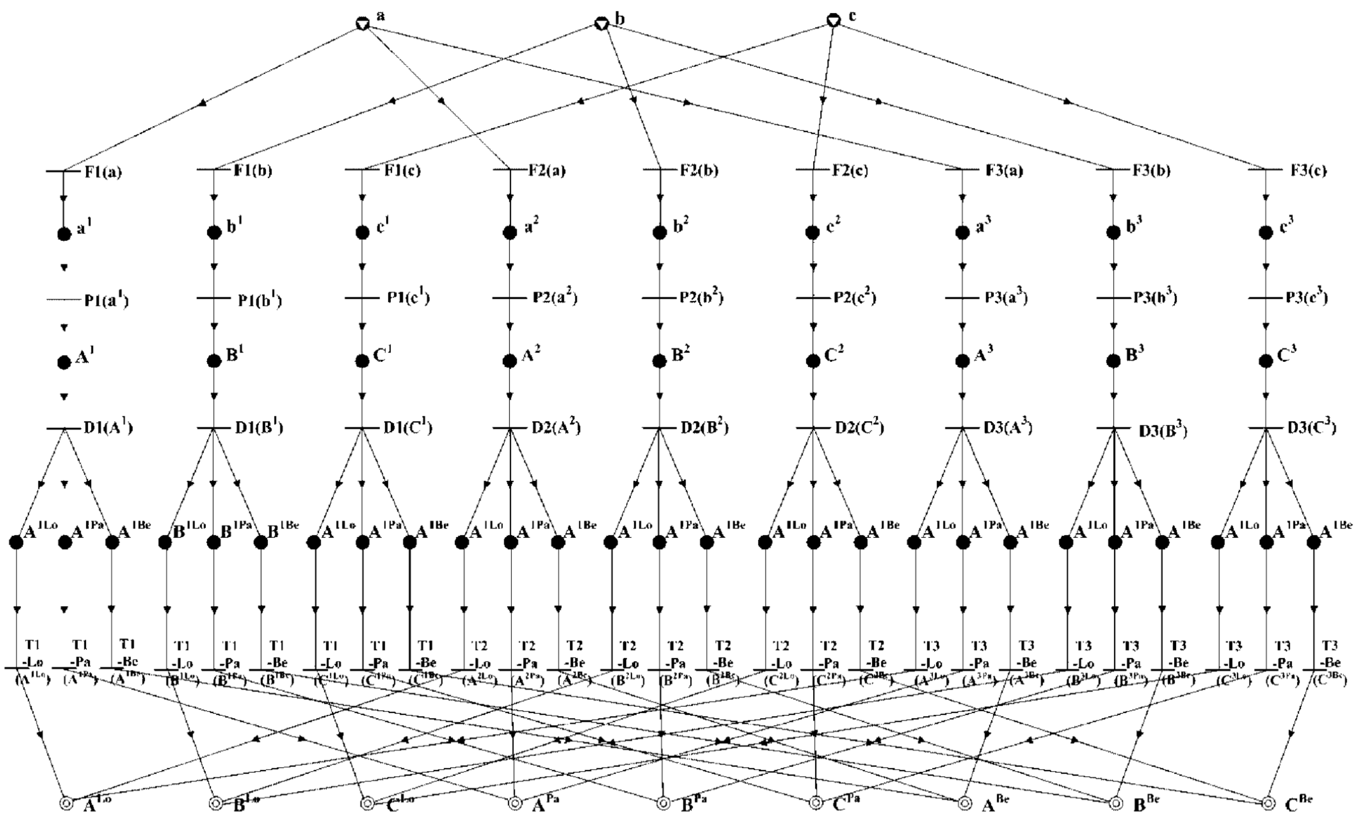


Figure 6. Maximal structure of the supply network in scenario II.

Table 3. Optimal and Near-Optimal Solutions Obtained by the MILP and the Graph-Theoretic Approach for Scenario II[a]

Supplying Quantity, $X_{l,d,i}$ (ton[b])		MILP			P-graph					
		Optimal			Optimal			2nd best		
		Lo	Pa	Be	Lo	Pa	Be	Lo	Pa	Be
plant 1	A	600	400	500	600	400	500	600	400	500
	B									
	C									
plant 2	A									
	B							400	400	600
	C	600	600	400	600	600	400			
plant 3	A									
	B	400	400	600	400	400	600			
	C							600	600	400
total profit ($/ton)			400 421			400 421			394 369	

[a] Notes: Scenario II corresponds to the coordination according to policy (a) in Ryu and Pistikopoulos.[4] [b] The quantities are interpreted on a unit time basis.

Table 4. Optimal and Near-Optimal Solutions Obtained by the MILP and the Graph-Theoretic Approach for Scenario III[a]

Supplying Quantity, $X_{l,d,i}$ (ton[b])		MILP			P-graph								
		Optimal			Optimal			2nd best			3rd best		
		Lo	Pa	Be	Lo	Pa	Be	Lo	Pa	Be	Lo	Pa	Be
plant 1	A	600			600				400			400	
	B	400			400				400			400	
	C	600			600				600			600	
plant 2	A		400			400				500	600		
	B		400			400				600	400		
	C		600			600				400	600		
plant 3	A			500			500	600					500
	B			600			600	400					600
	C			400			400	600					400
total profit ($/ton)			387 722			387 722			385 175			382 350	

[a] Notes: Scenario III corresponds to the cooperation according to policy (b) in Ryu and Pistikopoulos.[4] [b] The quantities are interpreted on a unit time basis.

C to plant 3, thus resulting in the total profit of 1.5% less than that of the optimal solution.

In scenario III of entailing each plant to supply a single market, the resultant optimal solution involves plant 1 supplying market Lo; plant 2, market Pa; and plant 3, market Be. In addition, the second-best solution involves plant 1 supplying market Pa; plant 2, market Be; and plant 3, market Lo, thereby yielding the total profit of 0.8% less than that of the optimal solution.

It is worth pointing out that the strategic requirements for product distribution are visualized as the operating (functional) units, that is distribution units without any cost, in scenarios II and III.

The importance of simultaneously generating the optimal and some near-optimal supply networks in the ranked order of the objective-function values cannot be overemphasized. These near-optimal networks serve as the stand-bys to immediately replace the optimal network in case of interruptions from man-made catastrophes, for example warfare, or natural catastrophes, for example earthquake. Such capabilities are totally void with the MIP. Moreover, one of the near-optimal solutions may be the optimal solution in practice because the mathematically optimal solutions might not be executable in reality.[14]

All computing has been performed on a PC (3.4 GHz Pentium D). The computing times for the current graph-theoretic method are less than 1 s for scenarios I and II, and 1.5 s for scenario III in simultaneously yielding the optimal as well as a finite number of near-optimal solutions. In contrast, the conventional algorithmic method resorting to mixed integer linear programming (MILP) yields the optimal but no near-optimal solutions for the three scenarios within 1 s. As for MIP, it is well-known that, in general, the computing time magnifies exponentially with the complexity of a network. It is also well-known that MIP often encounters difficulties in determining globally optimal solutions when the objective functions are nonlinear.[13] Such is not the case for the current graph-theoretic method due to its capability of exhaustively excluding combinatorially infeasible subnetworks in rigorously composing via algorithm MSG the maximal structure, which is the exact superstructure with minimum complexity. Moreover, the computing time for executing algorithm MSG increases polynomially instead of exponentially.

In light of the aforementioned unique attributes, it is highly likely that the proposed method will join the repertoire of other mathematical methods available for the planners or designers of supply chain networks. In general, however, any mathematical method provides only the benchmarks or frameworks, in terms of the boundary of the feasible region or the upper-bound or lower-bound of the objective function. Such boundary or bounds guide the configuration of realistic networks. This is akin to the role played by the ideal-gas law in describing the behavior of real gases. The synthesis of any realistic network entails the deployment of a variety of heuristics compiled by individual network planners. Furthermore, the values of specific parameters, which are characteristics of the network, need to be periodically and adaptively updated in response to situational variations, which can be environmental, economical, societal, and/or political.[14] Again, because of the aforementioned unique attributes of the proposed method, the parameters of the resultant network can be readily updated.

5. Conclusions

A novel paradigm is proposed for optimally designing an enterprise-wide supply network. It resorts to the efficient graph-theoretic method based on P-graphs (process graphs): In the broadest sense, a supply network is, in fact, a process network. The method simultaneously yields not only the optimal but also the near-optimal networks in the ranked order in terms of the objective function. The efficacy of the proposed paradigm is unequivocally demonstrated with an example involving three plants, three products, and three markets under various scenarios.

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Supporting Information Available: P-graph approach, algorithm MSG, algorithm SSG, algorithm ABB, parameters for the three scenarios, and operating units and materials in scenario II. This material is available free of charge via the Internet at http://pubs.acs.org.

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